Course Contents

MATH511 Quantum Mechanics (I) 3 credit hours

Foundations of Quantum Mechanics and its mathematical tools. Energy Spectra for some molecules. Wave Mechanics and Schrödinger equation. Scattering Theory.

MATH 512 Fluid Dynamics 3 credit hours

Fundamental concepts. Basic equation for incompressible flow. Navier-Stokes equations. Boundary Layer. Flow about an immersed body.

MATH 513 Perturbation Theory 3 credit hours

Asymptotic expansions, Regular perturbation problems, Methods used include matched asymptotic expansions, Lighthill's Strained coordinate technique and the method of multiple scale, Applications to problems in fluid Mechanics, Magnetohydrodynamics and Quantum Mechanics.

MATH 514 Calculus of Variations 3 credit hours

General variations of a functional constrained extrema. Euler equations. Hamilton-Jacobi equation and related topics. The second variation and sufficient conditions for an extremum.

MATH 520 Ordinary Differential Equations 3 credit hours

Existence and uniqueness of solutions of linear systems. Stability theory. Poincare's theory for two dimensional systems. Sturm-Liouville boundary problems.

MATH 521 Applied Differential Equations 3 credit hours

Partial Differential Equations as mathematical models of physical problems. Linear second order equations and their classification (Laplace's equation, wave equation, heat- equation). Methods of solution Green's function. Special analysis of elliptic differential operators in a Hilbert space.

MATH 522 Partial Differential Equations (I) 3 credit hours

The space of test functions $C_0^{\infty}(\Omega)$. The space of distributions and its topology. The convolution product of two distributions. Existence theorem for linear equations with constant coefficients. The space of tempered distributions and Fourier transforms. Sobolev spaces.

MATH 523 Partial Differential Equations (II) 3 credit hours

Treatment of the Theory of partial differential equations with emphasis on the fundamental features of elliptic equations. Existence and uniqueness of solutions for various types of boundary conditions. Discussion of representative examples of elliptic, parabolic and hyperbolic equations.

MATH 530 Introduction to Discrete Structures 3 credit hours

Graphs, Subgraphs, Trees, Connectivity, Euler Tours and Hamiltonian Cycles, Ordered Sets, Comparability and Covering Graphs, Dilworth Theorem, Block designs, Latin Squares, finite Geometries, Tournaments, Codes.

MATH 531 Graph Theory 3 credit hours

Colouring, Planar Graphs, Directed Graphs, Shortest Path Problem, Matching and b-matching, algorithms for Eulerian and Hamiltonian Walks, Independent Sets and Cliques, Graph Factorizations, Graph labellings.

MATH 532 Ordered Sets 3 credit hours

Fundamental theorems, algorithmic aspects of chains decompositions, cutsets, fibers, algorithmic and structural aspects of linear extensions, fixed points, the diagram, the dimension, the jump number, sorting, linear extensions and probability, many machine scheduling, order preserving maps, structure and classification.

MATH 533 Mathematical Logic 3 credit hours

The nature of mathematical logic (Axiom systems, Formal systems, Syntactical variables); First-Order theories (Functions and predicates, Truth functions, First Order languages, Structures, Logical axioms and rules); Theorems in First-Order Theories (the Tautology theorem, the Deduction theorem, the Equivalence and Equality theorems, Prenex form, Godel's Completeness theorem, Löwenheim-Skolem theorem).

MATH 534 Formal Languages and Complexity 3 credit hours

Deterministic automata; regular languages, context-free, Turing machines, Halting problem, The classes and NP, Cook's theorem (The NP-completeness of the satisfiability problem) , examples of NP-complete problems, complexity hierarchies.

MATH 535 Combinatorial Design 3 credit hours

Pairwise Orthogonal Latin Squares (POLS), Transversal Designs (TDs), Group Divisible Designs (GDDs), Pairwise Balanced Designs (PBDs), Room Squares, Balanced Incomplete Block Designs (BIBDs) , Finite Planes and Finite Geometries, Symmetric BIBDs,Methods of Direct and Recursive Constructions of Steiner Triple Systems (STSs) , Designs and Codes, Covering and Packing Designs.

MATH 536 Coding Theory 3 credit hours

Irreducible Polynomials on finite fields, algorithms of Berlekamp, single double-error correcting codes, cyclic codes, the group of a code and quadratic residue of a code.

MATH 537 Cryptography 3 credit hours

Introduction to cipher systems, Finite State Machines, Introduction to computational complexity, stream Ciphers, Cipher systems based on number theory.

MATH 540 Theory of Modules 3 credit hours

Modules and submodules, Isomorphism Theorems of Modules, Direct sum of modules, Projective modules, Injective modules, Exact sequences, Trosion modules, Free modules, Direct Decomposition of finitely generated modules over P.I.D. Application to group theory.

MATH 541 Group Theory I 3 credit hours

Structure of finitely generated abelian groups, Semi-direct product of groups, chain conditions, Free groups and presentation of groups.

MATH 542 Linear Algebra 3 credit hours

Linear functional and dual spaces, Canonical form of linear transformations, Jordan and rational forms, Multilinear forms, Hermitian, unitary and normal transformations, Tensor product of vector spaces.

MATH 543 Galois Theory 3 credit hours

Historical background. Separability and simple extensions. Galois extensions. Cyclotomic fields. Solvable and radical extension. Solvability of equations of degree less than five. Transcendental basis.

MATH 544 Ring Theory I — 3 credit hours

Hom and duality, Tensor product of modules, primitive rings, the Jacobson radical, prime radical, completely reducible rings, semisimple rings and certain relevant theorems, Artinian and Noetherian rings. On lifting idempotents, local and semi- perfect rings, The Brauer group.

MATH 545 Representation Theory of finite groups — 3 credit hours

Semi-simple modules, semi-simple rings, group Algebra representation, character, induced character, generalized character, Representation direct product, Representation of abelian groups, Clifford's theorem, applications.

MATH 546 Homological Algebra — 3 credit hours

Review of Modules (Tensor product of modules, the Horn functions, Free modules, projective and injective modules). Introduction to categories and functors, The homology and cohomology functors, cohomology of groups and its relation to extension problem.

MATH 547 Commutative Algebra — 3 credit hours

Ideals and their radicals, Modules, Noetherian and Artinian rings, Primary decompositions, localization, Principal ideal theorem, Cohen-Macaulay rings, Hilbert rings.

MATH 548 Fuzzy Algebraic Systems — 3 credit hours

Fuzzy sets, Fuzzy relations, Fuzzy subgroups, Fuzzy normal subgroups, Fuzzy congruences, Fuzzy ideals, Fuzzy prime and maximal ideals, Other Fuzzy substructures of algebraic systems.

MATH 549 Finite Fields — 3 credit hours

Minimal polynomials, Irreducible polynomials, automorphism groups of $GF(P^m)$, Primitive elements, Application of finite fields in designs of codes and aspects of cryptography like discrete logarithms and the use of elliptic curves.

MATH 550 Numerical Analysis — 3 credit hours

Norms, Arithmetic, and well-posed computations (Norms of vectors and matrices, Floating-point arithmetic and rounding errors, Well-posed computations); Iterative solution of non- linear equations(Functional iterations for a single equation: error propagation, second and higher order iteration methods. Some explicit iteration procedures: The Chord method, Newton method, method of false position and Aitkin's delta square method, Special methods for polynomials: evaluation of polynomials and their derivatives, sturm sequence, Bernoulli's method, Bairsou's method); Solution of Systems of Nonlinear equations: Substitution, Secant and Newton Raphson method, Continuation methods.

MATH 551 Numerical Linear Algebra — 3 credit hours

Direct solution of linear equations: Elimination and Factorization method, Ill-conditioning, Iterative refinement, Orthogonal Factorizations: (Jacobi, Gauss-Seidel, SOR, Conjugate Gradients, Pre-conditioning, Chebyshev semi-iteration methods). Matrix Eigenvalue Problems: Power method and inverse iteration, Jacobi, Givens and Householder methods, Sturm Sequence and QR method, Singular value decomposition.

MATH 552 Numerical Solution of Ordinary Differential Equation — 3 credit hours

Introduction: Taylor, Euler, and modified Euler methods. Linear Multistep Methods: Order, consistency, zero-stability, convergence, Bounds for local and global truncation error, Absolute and relative stability, Skob predictor-corrector methods, Milne's error estimate. Range-Kulta Methods: Derivation of classical RK methods of 2nd order, stability of RK methods. Boundary value problems: Finite difference methods, shooting methods, collocation method and variational methods.

MATH 553 Numerical Solution of Integral Equations — 3 credit hours

Review of basic theory of integral equations. Fredholm integral equations: Nystrom's method, product integration methods, projection methods, Eigenvalue problems, First Kind equation and regularization. Volterra Integral Equations: Quadrature, Spline methods and collocation. Integral equations of mathematical physics. Boundary Integral Equations.

MATH 554 Approximation Theory — 3 credit hours

Polynomial Interpolation: Lagrange interpolation formula, error in polynomial interpolation, Newton's interpolation method, Hermite interpolation. The approximation problem, existence of best approximation and uniqueness: approximation in a metric space, approximation in normed space, conditions for uniqueness of the best approximation, the uniform convergence of polynomial approximations, Least Squares approximation, Chebyshev approximation, Spline approximation.

MATH 555 Mathematical Programming — 3 credit hours

Simplex Method with its variant forms, Duality theory for linear programming, Sensitivity Analysis, Parametric programming, Integer programming, Goal programming, Applications in various fields.

MATH 556 Nonlinear Optimization Techniques — 3 credit hours

Search Methods for one variable, convex functional and their differentiability. Constrained problems: Jacobian method and Lagrangian technique, Kuhn-Tuker conditions. Unconstrained problems: Gradient method, quadratic programming, separable programming, Geometric programming, linear combination method, convex programming, Penalty methods and binar method.

MATH 570 Topology and Calculus in R^n 3 credit hours

Connected spaces, path connected spaces, components, locally connected spaces, Quotient spaces, separation axioms (Hausdorff, normal etc). Limits, continuity and differentiability of functions of several variables, Mean value theorem, Taylor's Theorem, Inverse and Implicit function theorems. Smooth manifolds, Tangent spaces, smooth functions on manifolds, Inverse and Implicit function theorems on manifolds.

MATH 571 Singular Homology and Cohomolyg Theories 3 credit hours

Singular Homology groups, Mayer-Vetories sequence, applications, attaching spaces with maps, CW-complexes, Cellular homology, Cohomology groups, Cup and Cap products, Duality on manifolds.

MATH 572 Vector Bundles and K-Theory 3 credit hours

General theory of Vector Bundles and K-Theory.

MATH 573 Differentiable Manifolds 3 credit hours

Definition and examples of manifolds, Submanifolds, tangent and cotangent bundles, Vector fields, Differential forms, Tensors, Integration on manifolds.

MATH 574 Geometry of Manifolds 3 credit hours

Differentiable manifolds. Tensor fields and operations. Differential forms and de Rham's Theorem. Principal fiber bundles, holonomy groups. Curvature form and structural equations. Bianchi's identity. Covariant differentiation. Geodesics. normal coordinates. Riemannian connection. Spaces of constant curvature. Schurs Theorem.

MATH 575 Geometry of Submanifolds 3 credit hours

Immersions, submanifolds of a Riemannian manifold, Gauss and Wenigarten formulae, Structure equations, Hypersurfaces in Euclidean space, Type number and rigidity, Minimal submanifolds with constant mean curvature, Total absolute curvature, tight immersions.

MATH 580 Measure Theory I 3 credit hours

Rings, Algebras, σ-algebras, Monotone classes, Measure: elementary properties, outer measure, extension, completion and approximation theorems, Lebesque's measure, Lebesque-Stielje's measure, Measurable functions, Integration with respect to a measure, the main theorems, the convergence of measurable functions, L^p spaces.

MATH 581 Functional Analysis I 3 credit hours

Normed spaces, Banach spaces, continuous linear operators, Hahn-Banach theorem, Duality, the open mapping theorem, The closed graph theorem, the uniform boundedness theorem, Hilbert spaces, Adjoint operators, unitary and normal operators, projections, spectral theory in finite dimensional spaces, spectral properties of bounded linear operators.

MATH 582 Functional Analysis II 3 credit hours

Compact linear operators and their spectral properties. Spectral properties of bounded, self-adjoint operators. spectral family of a bounded self-adjoint operator. Spectral representation of bounded self-adjoint operators. Banach Algebras. Spectral theory in Banach algebra. Commutative Banach Algebras. Gelfand Mapping. Spectral theorem for normal operators.

MATH 583 Topological Vector Spaces 3 credit hours

Filters, locally convex spaces. Linear maps. Quotient spaces, Normality, Metrizability, convergence of filters. Completeness. Locally compact spaces, finite dimensional spaces. Hahn-Banach-Dieudonne theorem. Grothendieke's completeness theorem.

MATH 584 Measure Theory (II) 3 credit hours

The product of two measure spaces. Fubini's theorem. Infinite product of probability spaces. Kolmogorov's consistency theorem. The Radon-Nikodym theorem. Conditional probability, conditional expectation. The Daniel's integral. Riesz- Representation Theorem. Haar measure on a compact group.

MATH 585 Complex Analysis 3 credit hours

Harmonic function, the general form of Cauchy's Theorem, Normal families, Conformal mapping. Analytic continuation, univalent function theory.

MATH 586 Potential Theory 3 credit hours

Harmonic and subharmonic functions in IR^n. Poisson integral. Classical Dirichlet problem. Different sets of axioms defining harmonic functions on a locally compact space. Superharmonic functions and potentials. Reisz-decomposition theorem for positive superharmonic functions. Balayage. Exceptional sets (e.g. polar sets, sets of capacity Zero), a convergence theorem for decreasing filtered superharmonic functions. Generalized Dirichlet problem in a harmonic space. Flux and its use in some superharmonic extension theorems in a harmonic space without positive potentials.

MATH 587 Summability Theory 3 credit hours

Some modes of convergence, general summability methods, some well known summability methods, Tauberian summability theorem.

MATH 588 Holomorphy and Calculus in Normed Spaces 3 credit hours

Multilinear Maps. Polynomials. Differential maps. Mean value theorem. Higher differentials. Finite expansion and Taylor's formula. Holomorphic functions. The strong Maximum Modulus theorem. Power series. Analytic mappings. Gateaux holomorphy. Radius of boundedness.