Course Contents

MATH5101 Ordinary Differential Equations 4(3+1)

Existence and uniqueness of solutions of linear systems. Stability Theory, Liapunov method. Two-dimensional autonomous systems, Poincare-Bendixson Theory. Second order linear differential equations: properties and zeros of the solutions. Sturm-Liouville theory: Linear differential operators in L^2 space, eigen functions and eigen values of self-adjoint linear operators, orthogonal polynomials and eigen function expansions.

MATH5121 Partial Differential Equations 4(3+1) Elementary theory of distributions. Types of linear partial differential equations. Fundamental solutions of some linear differential operators such as the Laplacian, the heat, and the wave operators. Applications to solving non-homogeneous problems. Green's function and its application to boundary-value problems. Elliptic equations: Boundary-value problems for the Laplacian operators in n-dimensions, harmonic functions, the Dirichlet problem, existence and uniquness of solutions. Generalizations and applications. Solutions of parabolic and hyperbolic equations in Sobolev spaces. Generalized mixed problems.

MATH5141 Quantum Mechanics 4(3+1) Foundation of quantum mechanics and its mathematical tools. Energy spectra for some molecules. Dirac formulation of quantum Mechanics (fundamental concepts), elementary quantum systems (application of wave mechanics and uncertainty principle). Symmetries in quantum mechanics (groups matrix and its generators), operator algebra. Quantum theory of radiation. Quantum theory of damping (Langevin approach).

MATH5301 Introduction to Discrete Structures 4(3+1)

Graphs, Directed graphs, Basic definitions, Isomorphism of graphs, Subgraphs, Paths and cycles, Matrix representation of graphs, Connectedness, Bridges, Cut-vertices. Trees, Spanning trees, Weighted graphs and minimum spanning trees, Shortest paths, Eulerian circuits, Hamiltonian cycles, Tournaments, Applications. Planar graphs, Euler's Formula, Kuratowski's theorem. Graph coloring, Vertex coloring, Edge coloring, Map coloring, Chromatic polynomials. Ordered sets, Comparability and covering graphs, Dilworth theorem, Block designs, Latin squares, Orthogonal Latin squares, Finite geometries, Basic definitions and properties.

MATH5311 Combinatorics 4(3+1)

General counting methods, The inclusion-exclusion principle, Ordinary generating functions, Exponential generating functions, Recurrence relations, Linear recurrence relations, Homogeneous recurrence relations, Nonhomogeneous recurrence relations, Polya counting theory, Equivalence relations, Permutation groups, Burnside's Lemma, Inequivalent colorings, Cycle index, Polya's enumeration formula. Mobius inversion formula, Techniques of computing Mobius functions, Mobius functions for special lattices. The Pigeon-hole principle and its generalizations, Ramsey numbers, Ramsey theorem .

Prerequisite (MATH 5301)

MATH5321 Ordered Sets 4(3+1)

Basic definitions, Fundamental theorems, Chain decomposition, Linear extensions, Fixed points, Algorithmic aspects of chain decomposition, Cutsets, Fibers, Cutest and fiber decomposition, Drawing, The diagram, Algorithmic and structural aspects of linear extensions, Dimension, Jump number, Sorting, Linear extensions and probability, Single machine scheduling, Many machine scheduling, Order preserving maps, Structure and classification, Lattices, Free Lattices, Distributive Lattices, Planar Lattices.

Prerequisite (Mat 5301)

MATH5391 Selected Topics in Mathematics 3(3+0)

The course covers selected topics in mathematics suggested by the student's supervisor.

MATH5401 Group Theory and Modules 4(3+1)

Group action on a set, Series of groups, Solvable groups, Supersolvable groups, Polycyclic groups and nilpotenl groups, Semi-direct product and group extensions, Free groups, group presentations, Finite and algebraic field extensions, Normal and separable extensions, Galois extensions, Galois group and Artin's Theorm.

MATH5411 Introduction to Rings and Modules 4(3+1)

Modules, Module homomorphisms, Exact sequences, External direct product, Internal direct product, Complete direct sum, direct sum, Free modules, Projective and injective modules, Modules over principal ideal domain, Algebras, Tensor products, Localization, Primary decomposition, Integrally closed domains, Chain conditions, Noetherian and Artinian rings. Prerequisite (MATH5401)

MATH5421 Rings and Modules 4(3+1)

Ring Extensions, Dedekind domains, Hilbert and Nullstellensatz Theorem. Simple and primitive rings, The Jacobson radical of a ring, Semi-simple rings, Wedderburn-Artin theorem for semi-simple Artinian rings, Essential and small submodules, Singular submodules, Radical of a module, Primitive rings and density theorem, prime ideals and lower nilradical. Prerequisite (MATH5401) and (MATH5411)

MATH 5431 Algebraic Number Theory 4(3+1)

Number fields, Solvable and radical extensions, Abel's theorem, Kummer theorem, The ring of algebraic integers, Trace and norm, Discriminant and integral basis, Prime factorization of ideals, Norm of ideals, Quadratic and cyclotomic fields, Transcendence Bases, Linear Disjointness and Separability.

Prerequisite (MATH5401) and (MATH5411)

MATH5501 Numerical Analysis 4(3+1)

Floating point arithmetic and rounding errors, well-posed computation and convergence. Numerical methods for solving nonlinear equations with one variable: bisection, regula-falsi, functional iterative, Newton, secant and Aitken Δ^2. Error and convergence analysis for these methods, Special numerical methods for solving polynomials: evaluation of polynomials and their derivatives, Matrix and vector norms, convergence of vectors, Method for solvingsystem of nonlinear equations: Fixed point, Newton, finite difference Newton, quasi-Newton, steepest descent, Error and convergence analysis for these methods

MATH5511 Numerical Linear Algebra 4(3+1)

Eigenvalues and eigenvectors. Special matrices. Direct methods for solving system of linear equations. Analyzing the errors involved using these methods. Iterative refinement method. Iterative methods for solving system of linear equations: Jacobi, Gauss-Seidel and SOR. Error and convergence analysis for these methods. Various methods for solving least square problems along with analytical and computational discussion. Numerical methods for the matrix eigenvalue problems: power and inverse power iteration, Jacobi, Givens, Householder, LR and QR. Singular value decomposition. Applications.

MATH5521 Numerical Solutions of Ordinary Differential Equations 4(3+1)

Multi-step methods for solving initial value problems in ODE: Euler, midpoint, trapezoidal, Simpson, Adam-Multon and other. Derivation these methods using Taylor expansion, integration and interpolation techniques. Error and convergence analysis for these methods: local and global errors, consistency and stability. Predictor corrector methods, error (Milne's device) and stability. Various Runge-Kutta methods: derivation of some of these methods. Error and stability of Runge-Kutta methods. Numerical solutions for solving system of first order ODE. Finite difference and shooting methods for solving linear and nonlinear boundary value problems in ODE. Error and convergence analysis for these methods. Applications.

MATH5701 Geometry and Topology 4(3+1)

Connected spaces, Path connected spaces, Connected components, Locally connected spaces, Quotient spaces, The separation axioms (Hausdorff, Regular, Normal). Differentiable manifolds, Submanifolds of IR^n and Classical Lie groups, Tangent spaces, Differentiable mappings between manifolds, Inverse and Implicit function theorems on manifolds.

MATH5711 Algebraic Topology 4(3+1)

Homotopy of paths, the fundamental group, The fundamental group of the circle, the punctured plane, S^n and surfaces, Covering spaces, lifting properties, The classification of covering spaces, universal cover and deck transformations, Chain complexes, simplicial homology, Homotopy invariance, Excision, Mayer-Vietoris sequence, Cellular homology.

MATH5721 Differential Geometry 4(3+1)

Definition and examples of manifolds, submanifolds, Immersions and submersions, Lie groups, Equivalence classes of curves and derivations, Tangent vectors, The tangent bundle of a manifold, Vector fields and flows, Lie derivatives and bracket, Differential forms, Integration on manifolds.

MATH5801 Measure Theory 4(3+1)

Rings, Algebra, σ-algebra, Monotone classes, Measure, elementary properties, outer measure, extension, completion and approximation theorems, Lebesgue's measure, Lebesgue-Stieltje's measure, measurable functions, integration with respect to a measure, the main theorems, the convergence of measurable functions, Radon-Nikodym theorem (absolutely continuous functions), Fubini-Tonelli theorem, L^P spaces: Holder and Minkowski inequalities, completeness of L^P spaces, L^P space as a Banach space, the dual of L^P space.

MATH5811 Functional Analysis 4(3+1)

Banach spaces: Basic properties and examples, convex sets, subspaces and quotient spaces, linear functional and the dual spaces, Hahn-Banach theorem, the uniform boundedness principle, the open mapping theorem and closed graph theorem, Hilbert spaces: the Riesz representation theorem, orthonormal bases, isomorphic Hilbert spaces, Operators on Hilbert spaces: Basic properties and examples, adjoints, projection, invariant and reducing subspaces, positive operators and the polar decomposition, self-adjoint operators, normal operators, isometric and unitary operators, the spectrum and the numerical range of an operator.

MATH5821 Complex Analysis 4(3+1)

Holomorphic functions, Cauchy-Riemann equations, power series, logarithmic function, Cauchy integral formula (general form), Analytic functions, zeros of holomorphic functions. Maximum principle, Liouville's theorem, fundamental theorem of algebra, open mapping theorem, Schwarz lemma, Mobius transformations, Rouche's theorem, Conformal mappings and Riemann's theorem, Topology on the space of holomorphic functions. Montel's theorem, Harmonic and subharmonic functions, Weirstrass's theorem, Mittag Leffler theorem, Introduction to several complex variables.

MATH5831 Advanced Functional Analysis 4(3+1)

Spectrum of an operator, compact linear operators and their spectral properties, spectral properties of bounded self-adjoint operators, positive operators, product of positive operators, square root of positive

operator, projection operators: Theorem (positivity, norm), Theorem (partial order), Theorem (product of projections), Theorem (sum of projections), Theorem (difference of projections), Spectral family of a bounded self-adjoint operator, Banach algebras, Gelfand' mapping, spectral theorem for normal operators.

MATH5991 Research Project 3(3+0)

The student undertakes a supervised independent study and review of current research papers in an active branch of Mathematics .