Doctorate Program

This Program has been implemented by the Department since the first semester of the year 1414 H.

Program Objectives

- Providing the student with a broad knowledge of mathematics particularly the in - depth knowledge of the chosen branch.
- Equipping the student with the ability to pursue independent research in an active area of mathematics.
- Meeting the requirements of higher educational institutions and research centers for highly qualified mathematicians.
- Satisfying the aspirations of a growing number of M.Sc. degree holders in mathematics to obtain a higher qualification locally.
- Promoting the quality of the Department's performance and fostering its research activities.

Admission Requirements

In addition to the admission criteria stipulated in the Unified Graduate Studies Regulations for Saudi universities, the applicant must satisfy the following requirements:

1) The applicant must hold an M.Sc. degree (Science or Education) in Mathematics with a GPA of at least 3.75 out of 5.

2) The applicant must score at least 83% in the Standardized Test of English Proficiency (STEP) or at least 500 in TOEFL.

3) The applicant must score at least 75% in the aptitude test for graduates organized by the National Center for Assessment in Higher Education.

4) The applicant must pass the personal interview.

Degree Requirements

1. The Study for the degree is by thesis and courses.

2. The student must successfully complete a minimum of 18 credit hours of which a maximum of six can be selected from the M.Sc. course content lists which he/she has not previously studied.

3. The student must pass a comprehensive examination to be held

accordance with the regulations and guidelines of the Graduate College. The examination must include Analysis, the specialization branch and a supporting branch. These branches are to be chosen with the consent of the Department Council.

4. The student must present a research thesis showing creativity and originality.

Study Plan

1. The Plan comprises 5 Tracks, namely;

- Track A: Algebra.
- Track B: Geometry and Topology.
- Track C: Analysis.
- Track D: Computational and Discrete Mathematics.
- Track E: Applied Mathematics.

2. The student must pass the common course MATH 690 (Advanced Topics in Mathematics).

3. Students at the same level in the same Track must study the same courses. These courses are assigned by the Department.

4. The Courses are taught in two semesters as follows:

First Semester

Three courses to be selected from the Track list and the M.Sc. lists (subject to stipulation 2 of the degree requirements).

Second Semester

Two courses from the Track list and MATH690 (Advanced Topics in Mathematics).

List of Courses

Track A: Algebra

MATH 641	Group Theory (II)	(3 hours)
MATH 642	Ring Theory (II)	(3 hours)
MATH 643	Algebraic Geometry	(3 hours)
MATH 644	Algebraic Number Theory	(3 hours)
MATH 645	Universal Algebra	(3 hours)

Track B: Geometry and Topology

MATH 671	Analysis on Complex Manifolds	(3 hours)
MATH 672	Variational Theory and Minimal Submanifolds	(3 hours)
MATH 673	Lie Groups and Symmetric Spaces	(3 hours)
MATH 674	Geometric Topology	(3 hours)
MATH675	Algebraic Topology	(3 hours)
MATH 676	Homotopy Theory	(3 hours)
MATH 677	Topology of CW-Complexes	(3 hours)

Track C: Analysis

MATH 681	Stochastic Differential Equations	(3 hours)
MATH 682	Ergodic Theory	(3 hours)
MATH 683	Complex Analysis (II)	(3 hours)
MATH 684	The Theory of Distributions	(3 hours)
MATH685	Harmonic Analysis	(3 hours)
MATH686	Function Algebras	(3 hours)
MATH 687	Geometric Function Theory	(3 hours)

Track D: Computational and Discrete Mathematics

MATH 631	Lattice Theory	(3 hours)
MATH 632	Coding Theory	(3 hours)
MATH 633	Enumerative Combinatorics	(3 hours)
MATH 634	Model Theory	(3 hours)
MATH 651	Numerical Analysis (II)	(3 hours)
MATH 652	Numerical Solutions of Partial Differential Equations	(3 hours)
MATH 653	M Theory of Integer Programming	(3 hours)
MATH 654	Dynamic Programming	(3 hours)
MATH 655	Variational Inequalities	(3 hours)

Track E: Applied Mathematics

MATH 611	Quantum Mechanics (II)	(3 hours)
MATH 612	Methods in Fluid Dynamics	(3 hours)
MATH 613	Topics in Deterministic and Non-deterministic Modeling	(3 hours)
MATH 652	Numerical Solutions of Partial Differential Equations	(3 hours)
MATH 684	The Theory of Distributions	(3 hours)

Course Contents

MATH 611 Quantum Mechanics (II)

Angular momentum, fundamental properties of Lie groups, the isospin groups, Quarks and SU3, Representations of the permutation group, Mathematical Excursion, field quantization, Quantum theory of relaxation, Quantum theory of scattering, the one center point interaction in three dimensions.

MATH 612 Methods in Fluid Dynamics

Basic equations of Compressible flow, Analytical and Computational methods of solving Navier-Stoke's equation, Boundary layer theory, Finite element methods for inviscid and viscous compressible flows.

MATH 613 Topics in Deterministic and Non-deterministic Modeling

Topics of interest in Mathematical Modeling.

MATH 631 Lattice Theory

Lattice Theory: Two Definitions of Lattices, Some algebraic concepts, Polynomials, Identities, and Inequalities, Free Lattices, Special elements, Distributive lattices: Characterization theorems, Congruence Relations, Boolean algebras, Topological Representation, Distributive lattices with pseudocomplementation, Modular and Semi-modular lattices: Modular lattices, semimodular lattices, partition lattices, complemented modular lattices, Equational classes of lattices.

MATH 632 Designs and Codes

Covering and Packing Designs and Codes, Skolem sequences and applications in Designs and Codes, Methods of finding Designs and Codes.

MATH 633 Enumerative Combinatorics

General counting methods, generating functions, recurrence relations, inclusion-exclusion principle, Polaya's enumeration formula, ordered sets, Mobius inversion formula, techniques for computing Mobius functions, Mobius functions for special lattices.

MATH 634 Model Theory

What is Model Theory?, Model Theory for sentential logic languages, models and satisfaction, theories and examples of theories, Elimination of quantifiers, Completeness and Compactness, Countable models of complete theories, Elementary extensions and elementary chains, Skolem functions and indiscernibles, Examples.

MATH 641 Group Theory (II)

Study of finite groups and infinite groups and their structures.

MATH 642 Ring Theory (II)

Study of some aspects of commutative and non-commutative rings.

MATH 643 Algebraic Geometry

Affine and projective varieties, Morphism, Rational maps, non-singular curves, Introduction to projective space, Riemann Roch's Theorem, Hurwicz's Theorem, embedding in projective space, Elliptic curves, classification of curves.

MATH 644 Algebraic Number Theory

Review of congruences and Chinese remainder theorem, Quadratic reciprocity, Dedekind domains, Integral ideals, Ideal class group, Norm and traces, Basis and discriminant computations, the arithmetic of number fields.

MATH 645 Universal Algebra

Concept of Lattices , Complete lattices, Equivalence relations and algebraic lattices, Closure Operators, Universal Algebras, Examples, Isomorphism algebras, Sub-algebras, Theorem of Birkhoff, congruences and Quotient Isomorphism Theorems, Direct products, Subdirect products, Varieties, Free algebras, Malcev conditions, Boolean algebras, Boolean rings, Ideals and Filters, Stone Duality.

MATH 651 Numerical Analysis (II)

Floating-point arithmetic and rounding errors: direct and iteration methods of solving systems of linear equations, Error estimates and convergence criteria. Iterative methods for the nonlinear operator equations: Fixed-point principle, Newton's Method, Kantorovich Method, Quasi-Newton's Method, Quasi-Newton's Method with error terms and estimates.

MATH 652 Numerical Solution of Partial Differential Equations

Finite Difference Methods: Elliptic, parabolic and hyperbolic equations, Accuracy, Consistency, Stability, Energy methods for stability, Hyperbolic systems, Boundary Conditions, Dissipation, Dispersion, Finite Element Methods: Elliptic equations, Ritz method, Galerkin method, Construction of basis function, Interelement continuity and patch test, Semi-discrete Galerkin methods for time dependent problems, Solution of systems of ordinary differential equations.

MATH 653 Theory of Integer Programming

Problems, algorithms and complexity, introduction to integer linear programming, Estimates in integer linear programming, the complexity of integer linear programming, totally unimodular matrices, recognizing total unimodularity, integral polyhedral and total dual integrality, Cutting planes, Further methods in integer linear programming.

MATH 654 Dynamic Programming

Basic theory relating to the functional equations of dynamic programming. Analytic and computational methods for one-dimensional and multi-dimensional problems, Lagrange multipliers and reduction of state dimensionality, Applications of dynamic programming in various fields.

MATH 655 Variational Inequalities

Basic Concepts. Formulation of the variational inequalities. Existence and Uniqueness results. Fixed point approach. Penalty method. Lagrange Multiplier Method. Error estimate for the finite element approximation. Applications. Linear Complementarity problems and its generalization. Equivalence among variational inequality problems. Unilateral problem and complementarity problem and their significance.

MATH 671 Analysis on Complex Manifolds

Vector Bundles, Almost Complex Manifolds, the Canonical Connection and Curvature of a Hermitian Holomorphic Vector Bundle, Sobolev Spaces, Differential Operators, Kaehler Manifold, Differential Operators on a Kaehler Manifold, The Hodge decomposition Theorem on compact Kaehler manifolds, Kodaira's vanishing theorem, Hodge manifolds.

MATH 672 Variational Theory and Minimal Submanifolds

The first and second variation of arc length, Jacobi fields conjugate points, comparison theorems of Morse and Rauch, Myer's theorem on compactness of Riemannian Manifolds, Variation of immersion, Normal variation, first and second variation of the area function, Minimal submanifolds, stability of minimal submanifolds, Index of minimal submanifolds, minimal submanifolds in spheres, complex submanifolds of a complex projective space.

MATH 673 Lie Groups and Symmetric Spaces

Lie Groups and their Lie-algebras, action of Lie groups on a smooth manifold, homogeneous spaces, Riemannian homogeneous spaces, the cannonical connection and Jacobi equations, two-point homogeneous spaces, Riemannian symmetric space, structure of orthogonal involutive Lie algebras, symmetric spaces and orthogonal involutive Lie algebras, curvature of symmetric spaces, Riemannian symmetric spaces of rank one.

MATH 674 Geometric Topology

Cohomology and duality theorems, deRahm's theorems, cup products and transversality theory of submanifolds.

MATH 675 Algebraic Topology

Extraordinary cohomology theories, K-Theory, fixed point theory.

MATH 676 Homotopy Theory

The fundamental problems: extension, homotopy, and classification, maps of the n-sphere into itself, filtered spaces, fibrations, homotopy and the fundamental group, spaces with base points, groups of homotopy classes, H-spaces, H's spaces, exact sequences of mapping functions; relative homotopy groups, the homotopy sequence, the operations of the fundamental group on the homotopy sequence, the Hurewicz map, the homotopy addition theorem, the Hurewicz theorems, homotopy relations in fiber spaces, fibrations in which the base or fiber is a sphere, elementary homotopy theory of Lie groups and their coset groups.

MATH 677 Topology of CW-Complexes

Cell-complexes, CW-complexes, homotopy properties of CW-complexes, cellular homology and cohomology.

MATH 681 Stochastic Differential Equations

Measure-theoretic background, Ito's integral, McShane's Integral, Ito's formula. Stochastic Differential Equations, existence and uniqueness of solutions, dependence on initial distributions, properties of solutions, solutions as Markov and diffusion processes, Generalization to Hilbert valued processes.

MATH 682 Erdogic Theory

Measure-preserving transformations. Recurrence. Ergodicity. The Ergodic Theorem. Mixing. Isomorphism and spectral invariants. Entropy. Bernoulli transformations. Topological Entropy.

MATH 683 Complex Analysis (II)

Mittag-Leffler Theorem. Weierstrass Theorem. Subharmonic functions. The domain of Holomorphy. Pseudoconvexity and plurisubharmonicity. Runge domain. The partial differentiation problem.

MATH 684 The Theory of Distributions

Test functions, semi-norms(locally convex spaces), the inductive limit topology of C_0^∞ (Ω), the topology on D' (Ω), the dual of C_0^∞ (Ω), the topology E'(Ω), the space of s(IR^n, Tempered distributions, The Fourier transformation in s(R^n), the Paly-Wienner-Schwartz theorem, The Sobolev spaces $H^p(R^n)$, some applications in partial differential equations.

MATH 685 Harmonic Analysis

Elements of the theory of topological groups, Integration on locally compact spaces, Invariant functional, Convolutions and group representation, Characters and duality of locally compact groups.

MATH 686 Function Algebras

Algebras of functions, the Silov boundary, Representations of the carrier space, Homomorphisms of certain function algebras into a Banach algebra, Direct sum decompositions and related results, completely regular commutative Banach algebras, the algebra C(Ω) for certain special Ω . The l_p -algebras, functions with absolutely convergent Fourier series, functions of class C(n), Continuous functions of bounded variation, holomorphic functions of one variable, algebra of power series.

MATH 687 Geometric Function Theory

Zeros of analytic functions, Rouches theorem, The open mapping theorem, The maximum principle, Schwarz's Lemma, Caratheodory's inequality concerning the real part of an analytic function, Conformal mappings, Reflexion Principles, Mobius transformations (a detailed study), Schwarz-Christoffel transformation, Riemann mapping theorem (without proof), Univalent functions, Koee's constant, a general discussion on Bieberbach's conjecture and the integral representation of univalent functions in a disc (without proof), Some special classes of functions: star-like, convex, typically real.

MATH 690 Advanced Topics in Mathematics

The course covers recent research topics in Mathematics.